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THE EFFECT OF STRAIN RATE ON THE CHARACTER OF $\boldsymbol{\sigma}-\boldsymbol{\varepsilon}$ DIAGRAMS

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An equation governing the evolution of elastic strains (stresses) for a given strain rate is obtained using the field theory of defects under the assumption of uniform defect distribution. Strain diagrams are constructed by numerical solution of this equation and qualitative analysis of the phase portrait of the corresponding dynamic system. The effect of strain rate on the mechanical properties of materials is studied.

The importance of investigation of the effect of strain rate on the mechanical properties of materials is motivated by the fact that during treatment and utilization, materials deform at different rates, from very low (for example, in creep) to very high (under shock loading) values. Some experimental results show that the mechanical properties of materials depend on strain or loading rate. These dependences are known for the hardness, strength, and yield point of materials [1–3]. In the present paper, the effect of strain rate on the mechanical characteristics of materials is for the model of a deformable body obtained using the field theory of defects [4–6]. The model is described by the system of dynamic equations

$$
\begin{gathered}
B(\nabla \cdot I) = -\boldsymbol{P}^{\mathrm{eff}}, \qquad \nabla \cdot \alpha = 0, \\
\nabla \times I = \frac{\partial \alpha}{\partial t}, \quad S(\nabla \times \alpha) = -B\,\frac{\partial I}{\partial t} - \sigma^{\mathrm{eff}}, \quad \frac{\partial \boldsymbol{P}^{\mathrm{eff}}}{\partial t} = \nabla \cdot \sigma^{\mathrm{eff}}.
\end{gathered}
\tag{1}
$$

Here α and I are the tensors of density and dislocation-flow density, respectively, σ^{eff} and $\boldsymbol{P}^{\mathrm{eff}}$ are the effective stresses and impulse, respectively, and B and S are constants. The cross and dot denote vector and scalar products, respectively. The effective stresses and impulse are determined by the total contribution of external and internal actions:

$$
\sigma^{\mathrm{eff}} = \sigma^{\mathrm{ext}} + \sigma^{\mathrm{int}}, \qquad \boldsymbol{P}^{\mathrm{eff}} = \boldsymbol{P}^{\mathrm{ext}} + \boldsymbol{P}^{\mathrm{int}}.
$$

According to [4, 5], the internal stresses and the impulse due to defects in a material can be expressed in terms of the characteristics of the defect field:

$$
\sigma^{\mathrm{int}} = S(\alpha \cdot \alpha - \delta\alpha^2/2) + B(I \cdot I - \delta I^2/2) + \eta I; \tag{2}
$$

$$
\boldsymbol{P}^{\mathrm{int}} = B(\alpha \times I). \tag{3}
$$

Expression (2) contains the term ηI proportional to the plastic-distorsion rate β^{pl}. The defect-flow density (see the definition in [7])

$$
I = -\frac{\partial \beta^{\mathrm{pl}}}{\partial t} \tag{4}
$$

models viscous stresses (by analogy with the models of a viscous liquid and a viscoelastic body [8]).

Various phenomenological methods for taking into account energy dissipation which lead to relation (2) are considered in [5, 9]. It should be noted that viscous stresses [8] imply the existence of a dissipative function that

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is quadratic with respect to the plastic-distorsion rate: $R = \eta_{ijkl} I_{ij} I_{kl}$ (η is the tensor of viscosity coefficients). At the same time, the classical theories of plasticity [10] deal with a dissipative function that is homogeneous with respect to the first power of plastic-strain rate: $R = \theta\sqrt{I_{(ij)} I_{(kl)}}$ (θ is a constant; the bracketed subscripts denote symmetrization). The last function describes the energy dissipation caused by nucleation of defects, and the corresponding friction force determines the limiting shear stress. In this case, it is assumed that energy is not dissipated during motion of defects. A quadratic dissipation function takes into account the energy dissipated during motion of defects and ignores the energy of defect nucleation. This is true for many materials in which microplastic deformation occurs [11].

Koneva and Kozlov [12] analyzed the evolution of dislocation structures and showed that if stresses are close to the yield point and defects are distributed randomly and do not form spatial structures, the defect distribution is uniform. In this case, from (1)–(3) we obtain the equation

$$B\,\frac{\partial I}{\partial t} + B\Big(I \cdot I - \frac{\delta}{2} I^2\Big) + \eta I + \sigma^{\text{ext}} = 0. \tag{5}$$

In the derivation of Eq. (5) governing the dynamics of the tensor of the defect-flow density, it was also assumed that the constant dislocation density is equal to zero (consequence of the conditions $\nabla\alpha = 0$ and $\partial\alpha/\partial t = 0$). Physically, this assumption is based on the results of [13], which show that plastic flow occurs owing to deformation defects formed in a loaded material. With allowance for the definition of the tensor of the defect-flow density (4) and the relation

$$\beta^{\text{pl}} = \beta^{\text{tot}} + \beta^{\text{el}}, \tag{6}$$

Eq. (5) for uniaxial deformation takes the form

$$B\,\frac{\partial^2 (E^{\text{tot}} - E^{\text{el}})}{\partial t^2} - \frac{B}{2}\Big(\frac{\partial (E^{\text{tot}} - E^{\text{el}})}{\partial t}\Big)^2 + \eta\,\frac{\partial (E^{\text{tot}} - E^{\text{el}})}{\partial t} - ME^{\text{el}} = 0, \tag{7}$$

where M is Young's modulus, β^{tot}, β^{pl}, and β^{el} are the total, plastic, and elastic distorsions, respectively, and E^{tot} and E^{el} are the corresponding strain components.

Given initial conditions, one can use Eq. (7) to analyze strain curves, from which many mechanical characteristics of materials can be determined. According to (7), for a constant strain rate ($\partial E^{\text{tot}}/\partial t = V = \text{const}$), we obtain

$$B\,\frac{\partial^2}{\partial t^2} E + (\eta - VB)\frac{\partial}{\partial t} E + \frac{B}{2}\Big(\frac{\partial}{\partial t} E\Big)^2 + ME = \eta V - \frac{B}{2} V^2, \tag{8}$$

where $E(0) = 0$ and $\partial E(0)/\partial t = V$ are the specified initial conditions and E is the longitudinal elastic strain. In the dimensionless variables $T = (\eta/B)t$, $a = (B/\eta^2)M$, and $b = (B/\eta)V$, Eq. (8) becomes

$$\ddot{E} + (1-b)\dot{E} + \dot{E}^2/2 + aE = b - b^2/2, \qquad E(0) = 0, \quad \dot{E}(0) = b \tag{9}$$

(the dot superimposed denotes differentiation with respect to time).

In addition to the numerical solution of (9), we consider results of qualitative analysis of the phase portrait of the two-dimensional dynamic system

$$\dot{x} = y, \qquad \dot{y} = -(1-b)y - y^2/2 - ax + b - b^2/2, \tag{10}$$

which is equivalent to system (9) for $x = E$ and $y = \dot{E}$. The stationarity condition ($\dot{x} = 0$ and $\dot{y} = 0$) implies that the system has a single singular point $y_0 = 0$, $x_0 = b(1 - b/2)/a$. To study the character of this singular point, it is necessary to determine the eigenvalues of the system. To this end, we write the coefficient matrix of system (10) linearized in the neighborhood of the singular point: $\begin{pmatrix} 0 & 1 \\ -a & b-1 \end{pmatrix}$. Solving the corresponding characteristic equation, we obtain the eigenvalues

$$k_{1,2} = (b-1)/2 \pm \sqrt{(b-1)^2 - 4a}\,/2. \tag{11}$$

Let the parameters a and b be positive. In this case, the singular point is not a saddle point. The character of the singular point is determined by the sign of the real part of the eigenvalues: for $b > 1$, a point moving along the trajectory recedes from the singular point, and for $b < 1$, it approaches the singular point.

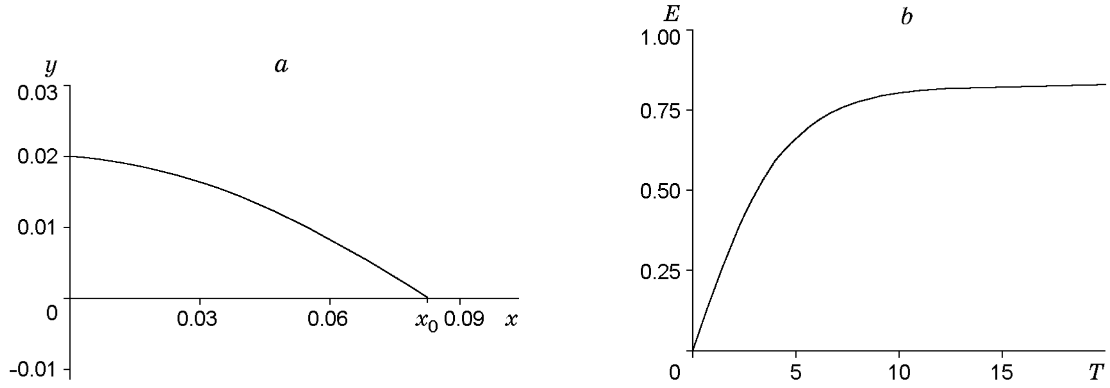

Fig. 1

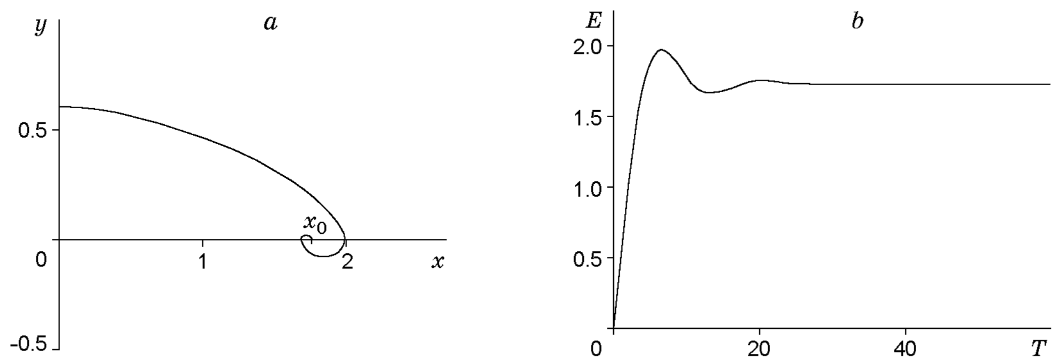

Fig. 2

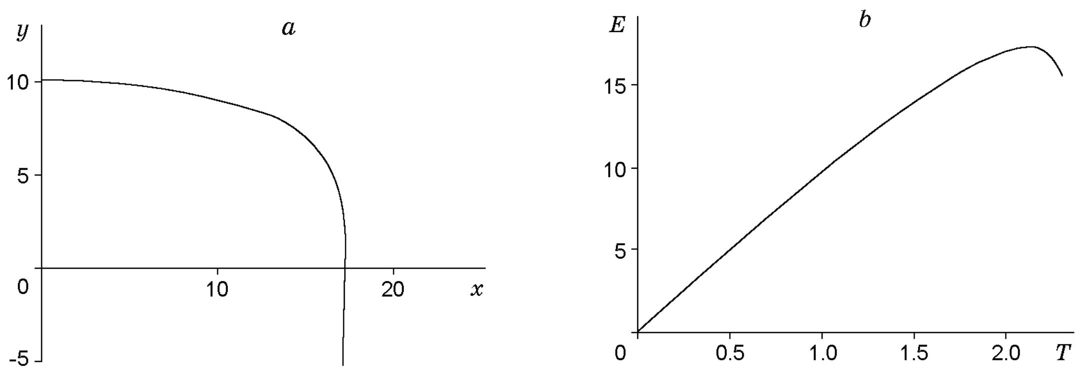

Fig. 3

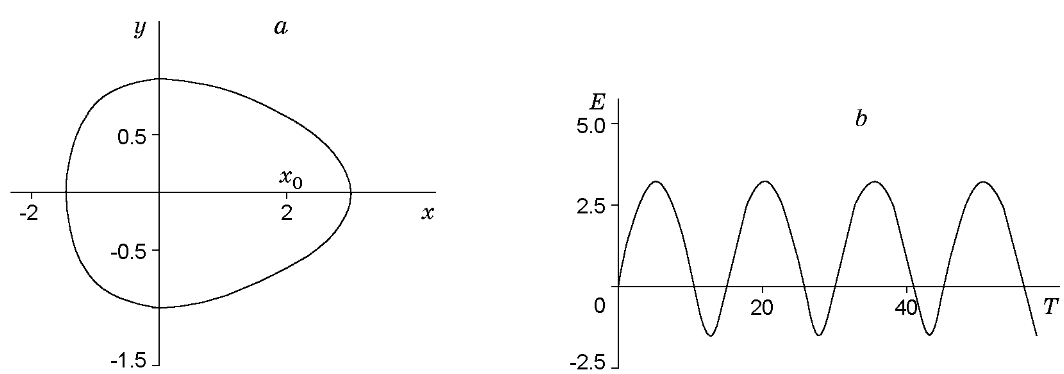

Fig. 4

We consider the case of $b < 1$. If $a < 1/4$, the character of the singular point changes as b increases from 0 to 1: it is a node for $0 < b < 1 - 2\sqrt{a}$, a degenerate node for $b = 1 - 2\sqrt{a}$, a focus for $1 - 2\sqrt{a} < b < 1$, and a center for $b = 1$. If $a > 1/4$, the singular point is a focus for any values of $0 < b < 1$. One can show that for a given initial condition [$E(0) = 0$ and $\dot{E}(0) = b$], the phase trajectory remains in the neighborhood of the singular point for $b \leqslant 1$ even if nonlinear terms are taken into account: $x = E \to x_0$ and $y = \dot{E} \to y_0$ as $T \to \infty$. From a physical viewpoint, this means that the elastic strain does not increase with time (its rate becomes equal to zero) and the total strain increases as a result of increase in plastic strain since these quantities are related by (6). As a matter of fact, an yield plateau is attained. The difference between a node and a focus is that in the first case, the trajectory reaches the limiting value immediately, whereas in the second case, there are exponentially attenuating oscillations about this value. Figure 1a and b shows characteristic phase trajectories and strain curves obtained for a node ($a = 0.24$ and $b = 0.02$) and Fig. 2 shows those obtained for a focus ($a = 0.24$ and $b = 0.6$). The strain curves are plotted in the coordinates "elastic strain – time" since for a constant strain rate, time is proportional to the total strain $T = E^{\rm tot}/b$.

We consider the case of $b > 1$. The singular point can be either a node or a focus. In both cases, a point moving along the trajectory recedes from the singular point. However, the type of singular point is of no importance for $b > 1$ since the leading part in the second equation (10) is now played by the nonlinear term $-y^2/2$, which "brings" the trajectory to infinity. In this case, the deformation curve corresponds to brittle behavior of the material. This case is shown in Fig. 3 ($a = 0.25$ and $b = 10$).

If the parameter b is close to unity, the real part of the eigenvalues (11) is small and oscillations about the singular point increase or attenuate very slowly. For $b = 1$, the singular point becomes a center; therefore, the phase trajectory is closed for the given initial conditions ($a = 0.24$ and $b = 1$ in Fig. 4). These oscillations are not observed in real systems. This implies that the model considered does not work in this region of loading rates. The following can be suggested. The equality $b = 1$ means that the loading rate V is determined by the relation $B/\eta = \tau$ [$V = \partial E^{\rm tot}/\partial t = (B/\eta)^{-1}$, where B and η are constants that characterize the inertia properties and viscosity of the ensemble of defects]. The quantity τ, which has the dimension of time, can be used as the relaxation time for the ensemble of defects [14]. The results obtained with the use of the model considered agree with experimental data for values $V \ll 1/\tau$ and $V \gg 1/\tau$, which ensure the validity of the initial assumption on uniform distribution of defects. For low loading rates, the system is able to relax to a state with uniformly distributed defects. For high loading rates, the initial uniform distribution of defects remains almost undisturbed owing to their inertia. In other words, in the first case, the defects are able to rearrange according to the external load, whereas in the second case, the external action has no time to disturb the system of defects. If the characteristic times are of the order of relaxation time, the uniform defect distribution can be disturbed, which cannot be described by this model.

In summary, the simplest model of a deformable body constructed with the use of the field theory of defects describes approximately the effect of strain rate on the mechanical properties of a material and allows one to infer how variation of these properties affects the deformation of solids. As was mentioned previously, the same material can exhibit plastic or brittle behavior at low and high rates, respectively [1]. Strain curves can be constructed for various materials deformed at low or high strain rates. An analysis of these curves shows that the coordinates of the singular point of the phase portrait that determines the yield point depend on strain rate, which also agrees with available experimental data.

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